Math 124
Final Exam V1

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Q1

Let the length of Pool $= x$

Width be $= y$

Given that it is twice as long as it is wide.

$\Rightarrow \quad y = 2x \quad -①$

Area of Pool $= xy = 1250$

Put the value of y from eq ①

$(x)(2x) = 1250$

$2x^2 = 1250 \quad \Rightarrow \quad x^2 = 625 \quad \Rightarrow x = 25m$

length $= 25$ meters

From eq ①

width $= y = 2 \times 25 = 50$ meters.

Q2. $2x^3 - 14x^2 + 3x - 21 = 0$

21 has a factor of 7, 3.

Checking whether $x = 7, -7, 3, -3$ is any of the factors of Polynomial

Putting $x = 7$

$\Rightarrow \quad 2(7)^3 - 14(7)^2 + 3(7) - 21$

$\Rightarrow \quad 686 - 686 + 21 - 21$

$= 0 \quad \Rightarrow LHS = RHS.$

$\Rightarrow x = 7$ is factor of Polynomial

To find remaining factors

$$x-7 \;\overline{)}\; 2x^3 - 14x^2 + 3x - 21 \;\overline{|}\; 2x^2 + 3$$

$$2x^3 - 14x^2$$

$$0 \quad +3x - 21$$

$$3x - 21$$

$$X$$

Here factor of Polynomial are $(x-7)(2x^2+3)$

Solution To check real solution.

$x - 7 = 0 \Rightarrow x = 7$ Real solution

$2x^2 + 3 = 0 \Rightarrow 2x^2 = -3$ Imaginay solution.

$x = 7$ is Real solution.

Q3 Bicyclist #2 takes 45 min to reach 12.5 miles
Bicyclist #1 takes 30 min to reach 12.5 miles

Head start should be $45 - 30 = 15$ minutes
to reach college at the same time.

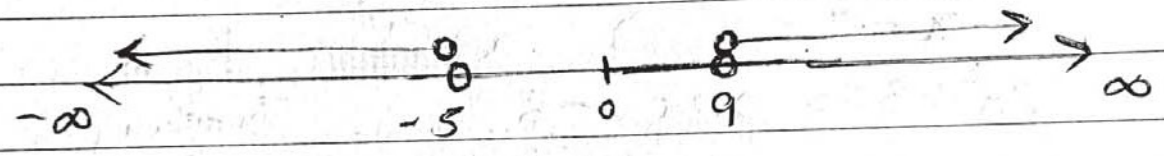
Q4.

$|x+3|-7>5$

$\Rightarrow |x+3|>5+7 \Rightarrow |x+3|>12$

$\Rightarrow x+3<-12$	$x+3>12$
$\Rightarrow x<-12-3$	$x>12-3$
$x<-15$	$x>9$
$\Rightarrow (-\infty,-15)$	$(9,\infty)$

Interval Notation $(-\infty,-15)\cup(9,\infty)$



Q5. $x^2+y-6=10$

y - Intercept $\Rightarrow x=0$

Putting eqn $(0)^2+y-6=10$

$\Rightarrow y=16$

y - Intercept = $[0,16]$

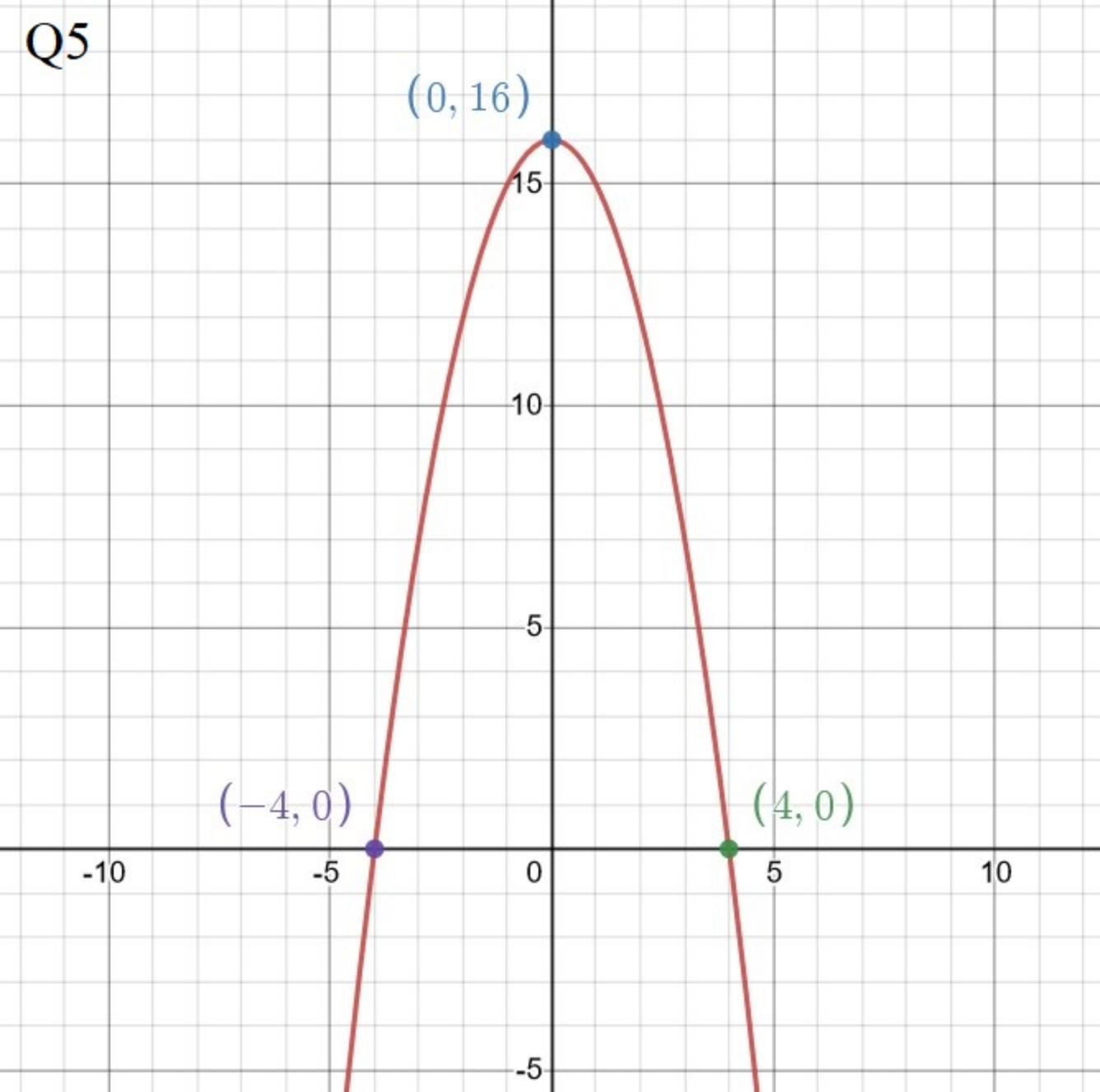
x - Intercept $y=0$

Putting in eqn $x^2+0-6=10$

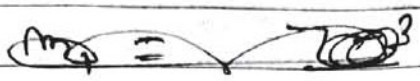
$x^2=16 \Rightarrow x=\pm 4$

x intercept $(4,0)$; $(-4,0)$

Q5



Q6 Slope of line 1 through (-2, 3) (0, 7)



$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{0-(-2)} = \frac{4}{2} = 2$$

Slope of line 2 through (-1, 5) and (1, 4)

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-5}{1-(-1)} = \frac{-1}{2}$$

Since $m_1 \times m_2 = 2 \times -\frac{1}{2} = -1$

Hence lines are Perpendicular.

Q7

a) From graph $f(1.2) = 3.1$

b) $f(x) = 3.1$; from graph we get $x = -3.1, 1.2$.

c) Domain from graph $[-3.1, 4]$

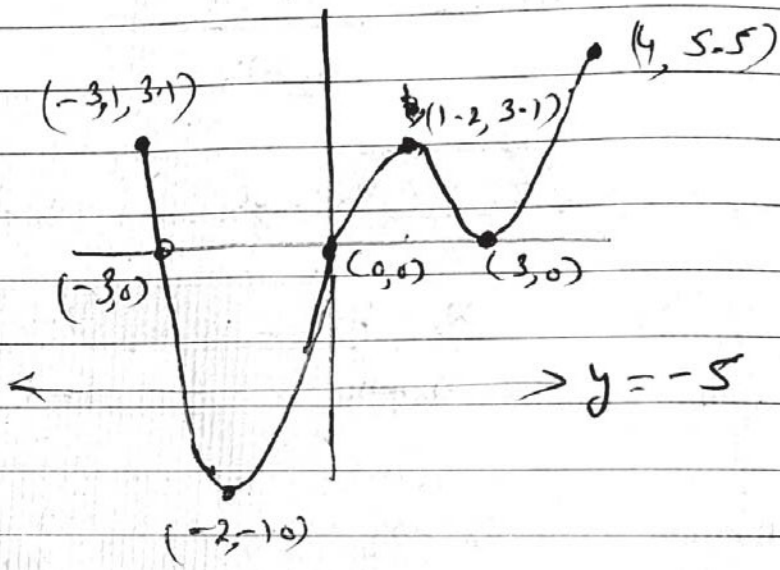
d) Range from graph $[-10, 5.5]$

e) Interval on which $f(x) \leq 0$

From graph $-3 \leq x \leq 0$

$[-3, 0]$

f) From graph



It can be clearly seen that $y = -5$ intersect curve 2 times.

Q8 a) From graph

Local Maxima = $(-3.1, 3.1)$; $(1.2, 3.1)$

b) Absolute Maxima = $(4, 5.5)$

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c) Local Minima = $(3, 0)$

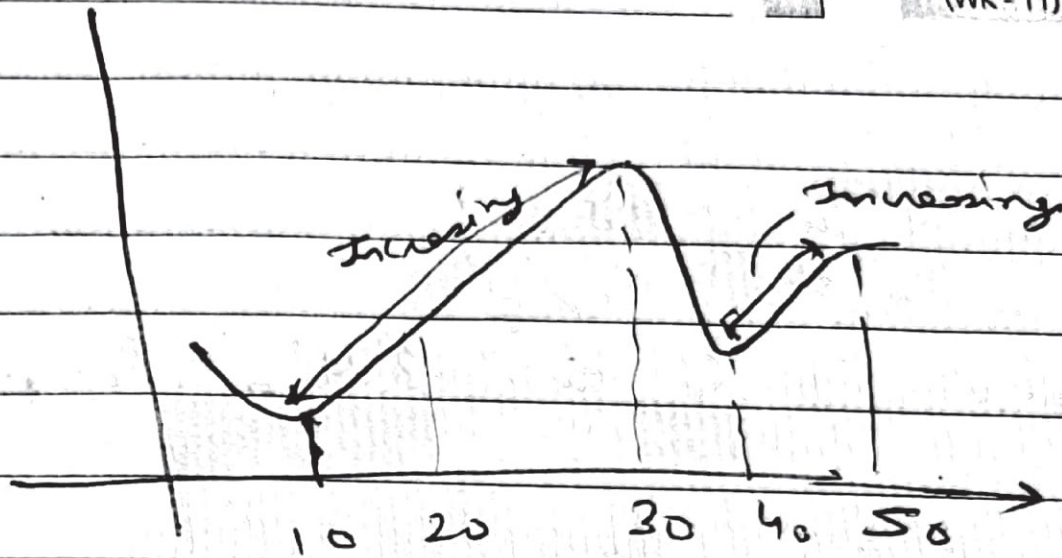
D) Absolute Minima = $(-2, -10)$

Q9.

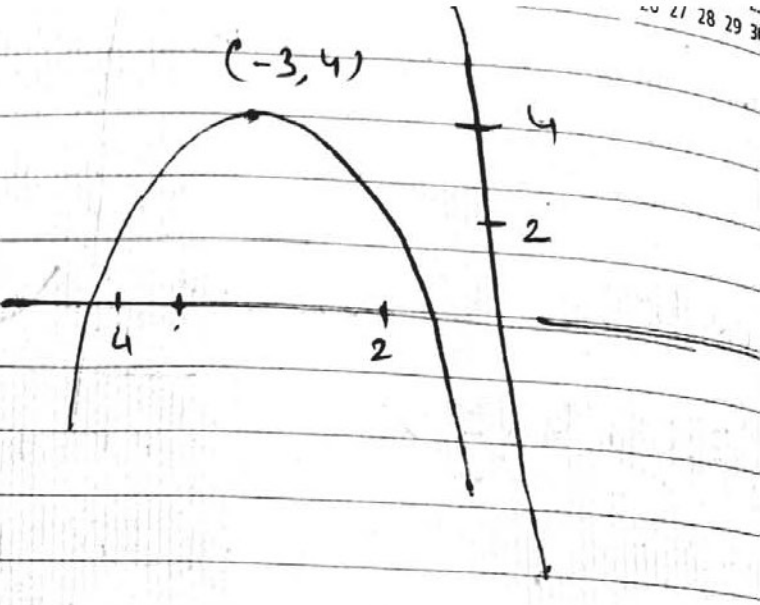
From graph

option (c)

(10, 30) U (40, 50)



Q10



From graph it is clear that Vertex of Parabola should be (-3, 4)

which is possible in Option (A) & Option (B)

It is a downward parabola

Hence Option (B) $f(x) = -(x+3)^2 + 4$

Q11 $f(x) = (x-2)^2(4-x)$

a) x - Intercept $\Rightarrow y = 0$

$\Rightarrow (x-2)^2(4-x) = 0 \Rightarrow x = 2, 4$

x intercept (2, 0) ; (4, 0)

b) y - intercept $\Rightarrow x = 0$

$\Rightarrow y = (0-2)^2(4-0) \Rightarrow 4 \times 4 = 16$

y - Intercept (0, 16)

c) For even multiplicity graph touch x-axis and for odd multiplicity graph crosses x-axis

$f(x) = (x-2)^2(4-x) = 0 \Rightarrow x = 2, 2, 4$

Zero	Multiplicity	Touch or crosses
2	2	Touch
4	1	Cross

d) degree of Polynomial = Number of roots = 3

e) Number of turning points = degree of Polynomial $-1 = 3-1 = 2$

Q12. a) For even multiplicity graph touch x-axis
for odd multiplicity graph crosses x-axis

a) even multiplicity $\Rightarrow$ touches

$x = 3$

b) odd multiplicity $\Rightarrow$ Crosses

$x = -3$

b) equation of f with min. degree of f possible

$f = (x-3)^2(x+3)$

$= (x^2-6x+9)(x+3)$

$= x^3-3x^2-9x+27$

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c) y - Intercept $\Rightarrow x = 0$

$\Rightarrow y = (0)^3 - 3(0)^2 - 9(0) + 27$

$= 27$

y - Intercept $(0, 27)$

Q13 $f(x) = y = \frac{x+2}{x-3}$

$\Rightarrow (y)(x-3) = x+2$

$xy - 3y = x+2$

$xy - x = 2+3y$

$x(y-1) = 2+3y$

$\Rightarrow x = \frac{2+3y}{y-1}$

$f^{-1}(x) = \frac{2+3x}{x-1}$

Q14 $5^{2x-1} = 625$

$\log_5 625 = 2x-1$

$4 = 2x-1$

$\Rightarrow 2x = 5$

$x = \frac{5}{2}$

Q15 $y = 3^x$

Inverse of $y = 3^x$

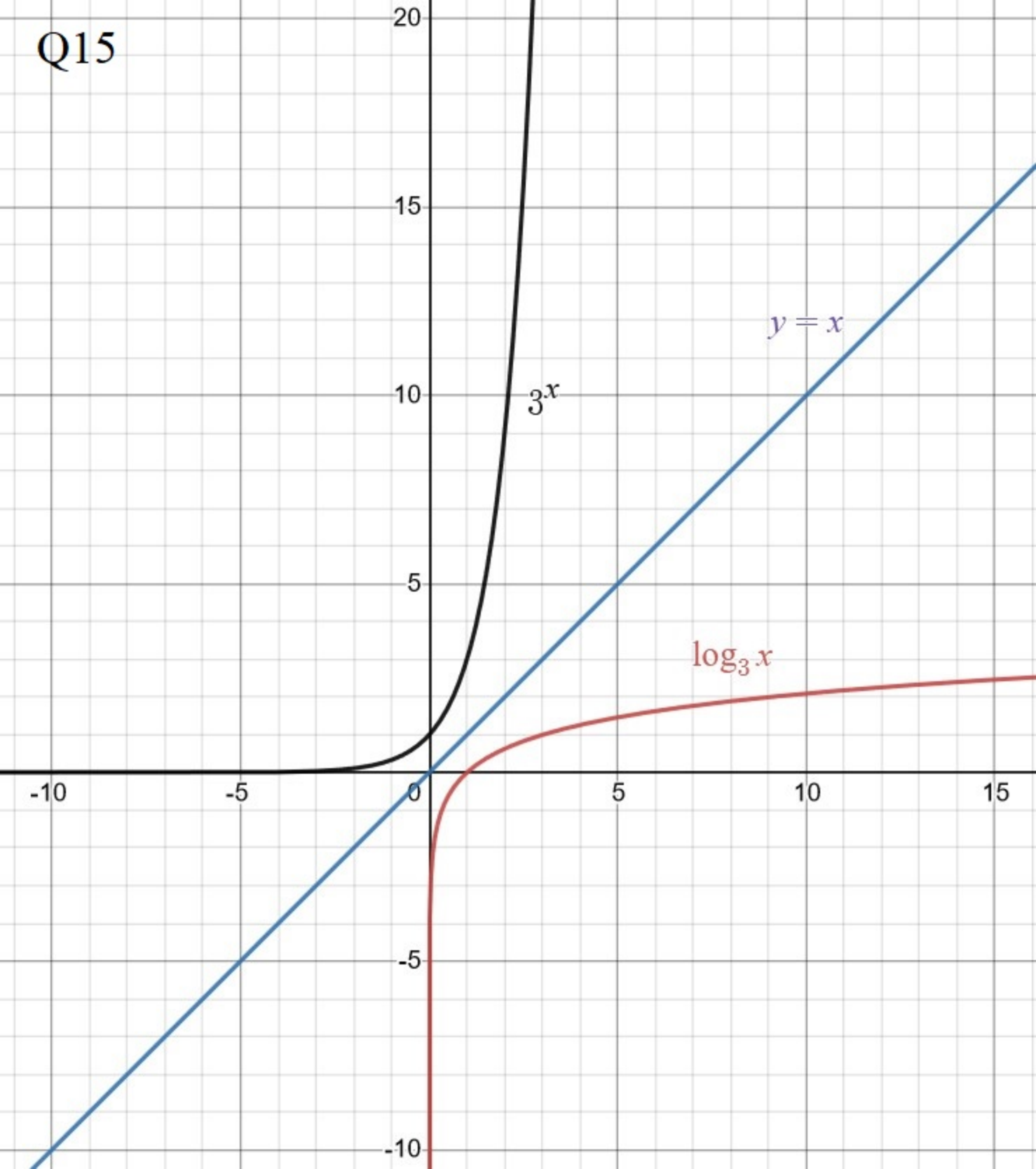
$$\log y = \log 3^x$$

$$\Rightarrow \log y = x \log 3$$

$$\Rightarrow x = \frac{\log y}{\log 3} = \log_3 y$$

$\Rightarrow$ Inverse $y = \log_3 x$

Q15



Q16 $\log_3 (x-5) = 4$

$(x-5) = 3^4$

$x - 5 = 81$

$\Rightarrow x = 86$

Q17 $\ln \dfrac{xy^3}{\sqrt{x^2-y}}$

$\Rightarrow \ln xy^3 - \ln \sqrt{x^2-y}$

$\ln x + \ln y^3 - \frac{1}{2} \ln (x^2-y)$

$\Rightarrow \ln x + 3 \ln y - \frac{1}{2} \ln (x^2-y)$

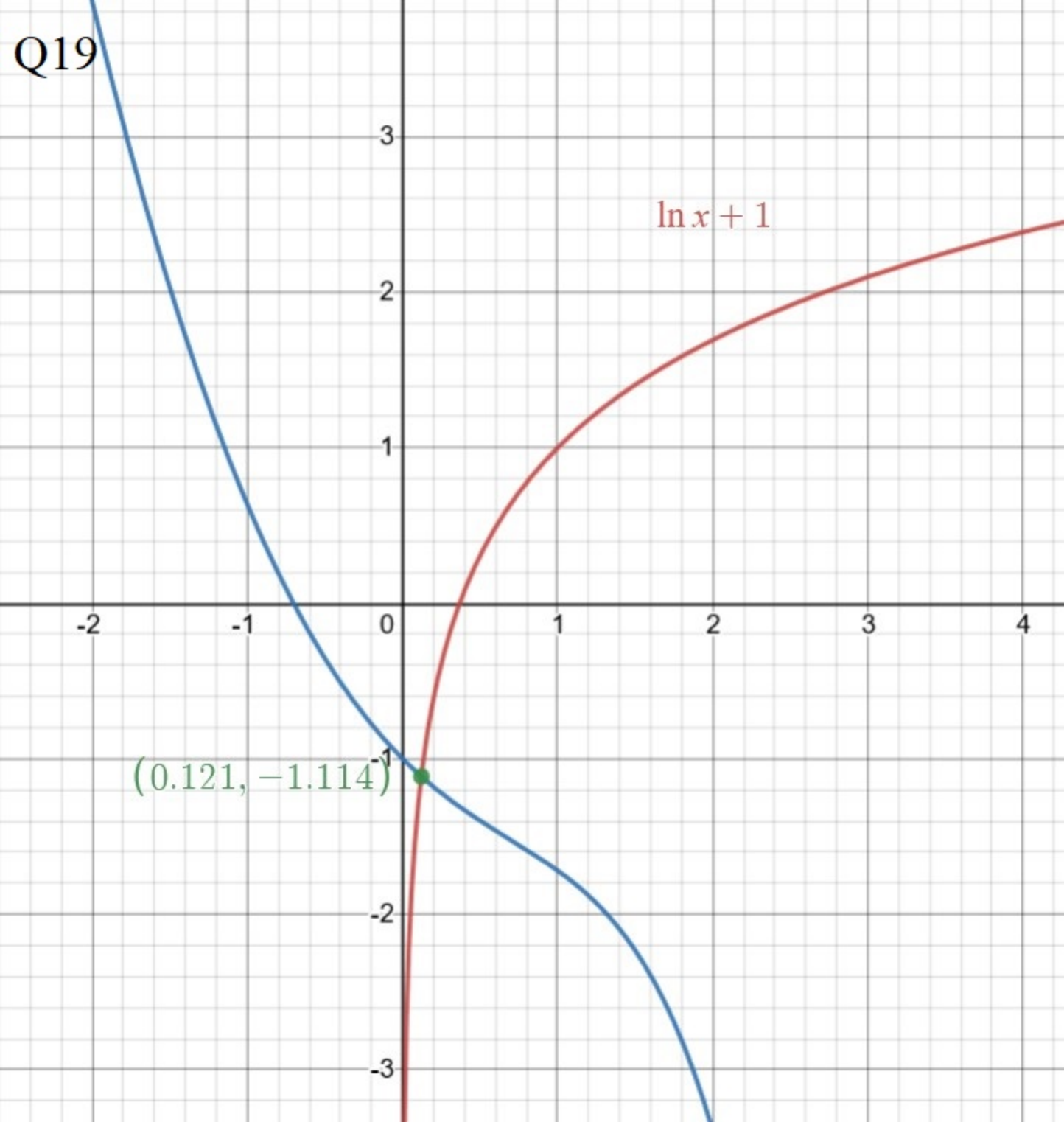
Q18 $3\log_2 x = -\log_2 27$

$\Rightarrow \log_2 x^3 = \log_2 (27)^{-1}$

$\log_2 x^3 = \log_2 \left(\frac{1}{27}\right)$

$\Rightarrow x^3 = \frac{1}{27}$

$\Rightarrow x = \frac{1}{3}$


Q19
3
2
1
-2
-1
0
1
2
3
4
-1
-2
-3
$\ln x + 1$
$(0.121, -1.114)$

P20

$$x + y + z = 9$$
$$2x + 2y - z = 6$$
$$3x - y + z = 7$$

Matrix $M = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & -1 \\ 3 & -1 & 1 \end{bmatrix}$; $S = \begin{bmatrix} 9 \\ 6 \\ 7 \end{bmatrix}$

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By Cramer rule.

$$M_x = \begin{bmatrix} 9 & 1 & 1 \\ 6 & 2 & -1 \\ 7 & -1 & 1 \end{bmatrix} ; \quad M_y = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 6 & -1 \\ 3 & 7 & 1 \end{bmatrix}$$

$$M_z = \begin{bmatrix} 1 & 1 & 9 \\ 2 & 2 & 6 \\ 3 & -1 & 7 \end{bmatrix}$$

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Determinant of Matrix M = D = -12

Determinant of Matrix $M_x = D_x = -24$

Determinant of Matrix $M_y = D_y = -36$

Determinat of Matrix $M_z = D_z = -48$

$$x = \frac{D_x}{D} = \frac{-24}{-12} = 2$$

$$y = \frac{D_y}{D} = \frac{-36}{-12} = 3$$

$$z = \frac{D_z}{D} = \frac{-48}{-12} = 4$$

$x = 2,\ y = 3,\ z = 4$

Q21

$$A = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 \\ -2 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} (5\times2)+(-1\times-2) & (5\times3)+(-1\times3) \\ (2\times2)+(3\times-2) & (2\times3)+(3\times3) \end{bmatrix}$$

$$AB = \begin{bmatrix} 12 & 12 \\ -2 & 15 \end{bmatrix}$$

Extra Credit

AP = 4, 7, 10, 13

$d = 7 - 4 = 3$

$a_1 = 4$

$a_n = a_1 + (n-1)d$

$a_{101} = 4 + (101-1)3$

$= 4 + (100)(3)$

$a_{101} = 304$

101^{st} term of AP = 304.